THE UNIVERSITY OF MELBOURNE

FACULTY OF BUSINESS & ECONOMICS

Helpsheet

Giblin Eunson Library

ARITHMETIC

Use this sheet to help you:

- Add, subtract, multiply and divide with positive and negative integers
- Use brackets to fine a common factor
- Add, subtract, multiply and divide fractions
- Convert fractions into percentages and vice versa
- Increase or decrease any number by a given percentage, and calculate percentage changes
- Understand and evaluate powers and roots
- Convert numbers to and from scientific notation

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ARITHMETIC

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Arithmetic Symbols

≠ means 'is not equal to'
≈ means 'is approximately equal to'
≡ means 'is identically equal to'
• means 'infinity'. (Note that infinity is not 'a very large number'. Any number, however large, is finite; while the meaning of infinity is that it is not finite).
-• means 'minus infinity'
$|x|$ means the absolute value of x: i.e., ignoring its sign. E.g. $|-2|$ = 2.
> means 'greater than'
< means 'less than'
=> means 'implies'. E.g., x = 5 => x + 3 = 8

Order of Operations

When working out any mathematical expression which contains more than one operation, the operations **must** be carried out in a particular order. The order is
B E D M A S – **B**rackets **E**xponents **D**ivision **M**ultiplication **A**ddition **S**ubtraction.

Exponents are terms written in 'index or power form'.
'Of' is equivalent to multiplication.
e.g. 4 x (5 + 6 -1)

Applying **BEDMAS** rule, this is evaluated as 4 x 10 = 40
Another way to solve it is to multiply it out as follows:
4 x (5 + 6 – 1) = (4 x 5) + (4 x 6) – (4 x 1)
= 20 + 24 – 4
= 40 (as before)

If there is a one set of brackets inside another set of brackets, start with the innermost brackets and work outwards

e.g. [(42 + 6) ÷ (5 + 1)] ÷ 2
= [48 ÷ 6] ÷ 2
= 8 ÷ 2
= 4

Multiplication by negative numbers

Care needs to be taken when multiplying by a negative number.
A **negative** multiplied by a **positive** results in a **negative**. Eg. -2 x 6 = -12

A **negative** multiplied by a **negative** results in a **positive**. Eg. -2 x -6 = 12

Factorisation

The process of factorisation involves finding a **common factor** of two or more terms and then taking this common factor outside a pair of brackets.

e.g. factorise 15 + 18

On inspection both numbers are divisible by 3, therefore 3 is a **common factor** of 15 and 18.

So we can write
15 + 18 = (3 x 5) + (3 x 6)
= **3 x (5 + 6)** (factorised form)

Expansion

The process of expansion is the opposite of factorisation ie. When given an expression in factorised form you are required to multiply out (expand) the brackets.
e.g. 3 x (5 + 6)
= (3 x 5) + (3 x 6)
= **15 + 18** (expanded form)

Fractions

a/b a = numerator; b = denominator; / = division

To **simplify** fractions you need to divide both the numerator and the denominator by a common factor. This process is called **cancelling** down.

e.g $\frac{4}{16} = \frac{(4 \div 4)}{(16 \div 4)} = \frac{1}{4}$

*As long as both the numerator and denominator are divided (or multiplied) by the same number then the value of the fraction has not altered.

To **add** or **subtract** fractions a **common denominator** needs to be found. Once two or more fractions have the same denominator then the numerators can be added or subtracted – the denominator will remain the same, i.e. it is **not** added or subtracted.

e.g. $\frac{2}{3} + \frac{3}{5} = \frac{2 \times 5}{3 \times 5} + \frac{3 \times 3}{5 \times 3}$

$= \frac{10}{15} + \frac{9}{15}$

$= \frac{19}{15}$

To **multiply** fractions, multiply the two numerators together and multiply the two denominators together.
e.g. $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$

When two fractions are multiplied and the result is 1, then they are said to be reciprocals.

e.g. $\frac{3}{4} \times \frac{4}{3} = \frac{3 \times 4}{4 \times 3} = 1$ $\frac{3}{4}$ and $\frac{4}{3}$ are each others **reciprocals**.

To **divide** by a fraction, invert the fraction you are dividing by and then multiply instead of divide. i.e. multiply the fraction by the reciprocal of the fraction you are dividing by.

e.g. $\frac{3}{5} \div \frac{4}{7} = \frac{3 \times 7}{5 \quad 4} = \frac{21}{20}$

Decimals

A decimal fraction is just another way of expressing a fraction in which the **denominator** is 10, 100, 1000 etc. The non-zero digits after the decimal point give us the **numerator** of the fraction, while the number of digits after the decimal point gives us the number of zeros in the **denominator**.

e.g $0.37 = \frac{37}{100}$

To **multiply** a decimal by a power of 10, then the decimal point is moved a number of places to the right by the number the power is, so if multiplied by 10 (10^1) then it is moved 1 place to the right, by 100 (10^2), 2 places and so on.

To **divide** a decimal by a power of 10, then the decimal point is moved a number of places to the left by the number the power is, so if multiplied by 10 (10^1) then it is moved 1 place to the left, by 100 (10^2), 2 places and so on.

Percentages

Roughly translated percent means 'out of a hundred'. Therefore 30% means '30 out of every 100' which can also be written as $\frac{30}{100}$.

So, in general a number written in percentage form is the same thing as the numerator of a fraction in which the denominator is 100. We avoid mixing decimals with fractions, so if the expression is 2.5% we would write this as $\frac{2.5}{100} = \frac{25}{1000}$ which would then be written as $\frac{1}{40}$

Converting a fraction into a percentage and a percentage into a fraction

To convert a fraction to a percent **multiply** by 100.

e.g. ⅛ x 100 = $^{100}/_{8}$ = 12.5%

To convert a percentage into a fraction **divide** by 100

e.g. 12.5% = $^{12.5}/_{100}$ = $^{125}/_{1000}$ = ⅛

The same rules apply for conversion of decimals to percentages and vice versa.

e.g 1: 3.4 x 100 = 34%

2: 125% = 125 ÷ 100 = 1.25

Finding a given percentage of an number

Change the percentage to a fraction and multiply by the number.

e.g. 20% of 90 = $^{20}/_{100}$ x 90 = $^{1800}/_{100}$ = 18.

Increasing a number by a given percentage

Suppose a dress costs $90 before GST at 10% has been added. What is the price including GST?
Intuitively we can see that 10% of $90 is $9, so the price including GST is $99.
From this example we can derive a general rule:
Price including GST = price before GST x 110%
= price before GST x 1.1.

Decreasing a number by a given percentage

The above rule can be reversed to work back from the price including GST to the price before GST. Instead of multiplying by 1.1, divide by 1.1.
e.g. A mobile phone cost $176 including GST? What is the price before GST is added?
$^{176}/_{1.1}$ = 160, so the cost of the mobile phone before GST is $160.

Powers and Roots

Squares
5^2 (5 x 5 = 25): **5** is the base and is the **power** or **exponent**.
Fractions can also be raised to a power.

*Remember when squaring a negative number the result will be positive.

Square Roots
The reverse of squaring a number is to find its **square root**.
$\sqrt{25}$ = 5 because 5^2 = 25, however $(-5)^2$ =25 also, so $\sqrt{25}$ has two square roots: 5 and -5 or ± 5.
The square root of a negative number
(e.g.$\sqrt{-25}$) does not exist.

Cubes and higher powers
6^3 (read as 6 cubed or 6 to the power 3) = 6 x 6 x 6 = 216.

*When cubing a negative number the result will be negative.

Any number can be raised to higher powers
e.g. 4^5 = 4 x 4 x 4 x 4 x 4 = 1024

Cube and higher roots
The reverse of cubing a number is to find its cube root.
$\sqrt[3]{216}$ = 6 because 6^3 = 6 x 6 x 6 = 216.

We can also find the cube root of a negative number.

e.g. $\sqrt[3]{-216} = -6$ because $(-6)^3 = -6 \times -6 \times -6 = -216$.

Higher roots can also be found.

e.g the fifth root of 1024 ($\sqrt[5]{1024}$) is 4.

Negative powers

Negative powers denote fractions.

e.g. $2^{-3} = 1/2^3 = 1/8$.

*Any number raised to the power 0 equals 1.

e.g. $7^0 = 1$.

Standard Index Form

Standard index form is called scientific notation. It is a way of writing very small or very large numbers with less risk of error or misreading. In scientific notation, any number can be written as a number between 1 and 10, multiplied by 10 raised to some power (either positive – for a large number; or negative – for a small number).

e.g. 1: $4876 = 4.876 \times 10^3$

2: $0.0008457 = 8.457 \times 10^{-4}$