M

THE UNIVERSITY OF MELBOURNE

FACULTY OF BUSINESS & ECONOMICS

Helpsheet

Giblin Eunson Library

FUNCTION DERIVATIVES

Use this sheet to help you:

- Find derivatives using the main rules of differentiation

Authors: Davies, W. M. and Beaumont, T. J.
Design and Layout: Pesina, J.

Rules for evaluating the derivative of a function

The process of finding a derivative is known as differentiation. The rules for differentiating speed up the process for finding the derivative enormously.

	Function	**Rule**
1.	y = axn $y=dx^n$	$\frac{dy}{dx} = a\,nx^{n-1}$
2.	y = c	$\frac{dy}{dx} = 0$

3. The chain rule

y = f(g(x)), let g(x) = u $\qquad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

where $\frac{dy}{du}$ is the derivative of y = f(u), and $\frac{du}{dx}$ is the derivative of u = g(x)

4. The product rule

y = uv where u and v are functions of x $\qquad \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ where $\frac{du}{dx}$ and $\frac{dv}{dx}$ are the derivatives of u and v.

5. The quotient rule

$y = \frac{u}{v}$ where u and v are functions of x $\qquad \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Examples:

1 (a) If $y = x^2$, we have n = 2, so $\frac{dy}{dx} = 2x^{2-1} = 2x^1 = 2x$

(b) If $y = x^{½}$, we have n = ½, so $\frac{dy}{dx} = ½x^{½-1} = ½x^{-½}$

*remember that $x^{½}$ is the same thing as $\sqrt{x}$.

(c) If $y = x^{-2}$, we have n = -2, so $\frac{dy}{dx} = -2x^{-2-1} = -2x^{-3}$

*remember that x^{-2} is the same thing as $\frac{1}{x^2}$, and $-2x^{-3}$ is the same as $\frac{-2}{x^3}$

(d) If $y = 10x^3$, we have a = 10 and n = 2,

$\frac{dy}{dx} = 10 \text{ x } 3x^{3-1} = 30x^2$

If $y = x^2 + 6$, we have n = 2 and c = 6,

so $\frac{dy}{dx} = 2x^{2-1} + 0 = 2x$

2 (a) If $y = (x^2 + 5x)^3$, we need to apply the chain rule, so let u = x

$y = u^3$, where $u = x^2 + 5x$.

This gives us $\frac{dy}{du} = 3u^2$ and $\frac{du}{dx} = 2x + 5$, so $\frac{dy}{dx} = 3u^2(2x + 5)$

Now replace u with $x^2 + 5x$ and get

$\frac{dy}{dx} = 3(x^2 + 5x)^2 (2x + 5)$

(b) If $y = (x^2 + 1)^5$, we have $y = u^5$ where $u = x^2 + 1$.

So $\frac{dy}{du} = 5u^4$ and $\frac{du}{dx} = 2x$, therefore $\frac{dy}{dx} = 5u^4(2x)$

Now replace u with $x^2 + 1$ and get $\frac{dy}{dx} = 5(x^2 + 1)^4 (2x)$

which simplifies to $\frac{dy}{dx} = 10x(x^2 + 1)^4$

3 (a) If $y = (x^2 + 1)(x^3 + x^2)$, we need to apply the product rule, $u = x^2 + 1$ and $v = x^3 + x^2$.

Now we have $y = u.v$

$\frac{du}{dx} = 2x$ and $\frac{dv}{dx} = 3x^2 + 2x$, and

$\frac{dy}{dx} = (u)(3x^2 + 2x) + (v)(2x)$

Now, replacing u with $x^2 + 1$ and v with $x^3 + x^2$, we get

$\frac{dy}{dx} = (x^2 + 1)(3x^2 + 2x) + (x^3 + x^2)(2x)$

Multiplying out the brackets and simplifying

$\frac{dy}{dx} = 3x^4 + 3x^2 + 2x^3 + 2x + 2x^4 + 2x^3 = 5x^4 + 4x^3 + 3x^2 + 2x$

(b) If $y = (5x^2 + 3x)(x^4 + x)$, applying the product rule, we get $y = u.v$, where $u = 5x^2 + 3x$ and $v = x^4 + x$.

$\frac{du}{dx} = 10x + 3$ and $\frac{dv}{dx} = 4x^3 + 1$

so $\frac{dy}{dx} = (5x^2 + 3x)(4x^3 + 1) + (x^4 + x)(10x + 3)$

After multiplying out the brackets and simplifying we get

$\frac{dy}{dx} = 30x^5 + 15x^4 + 15x^2 + 6x$.

4 (a) If $y = \frac{x^2 + 1}{x^3 + x^2}$, we need to apply the quotient rule, where $u = x^2 + 1$ and $v = x^3 + x^2$. Now we have $y = \frac{u}{v}$.

Find the derivatives of u and v.

$\frac{du}{dx} = 2x$ and $\frac{dv}{dx} = 3x^2 + 2x$

Applying the quotient rule $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

we get, $\frac{dy}{dx} = \frac{(x^3 + x^2)(2x) - (x^2 + 1)(3x^2 + 2x)}{(x^3 + x^2)^2}$

multiplying out the brackets

$\frac{dy}{dx} = \frac{2x^4 + 2x^3 - (3x^4 + 2x^3 + 3x^2 + 2x)}{x^6 + 2x^5 + x^4}$

$= \frac{-x^4 - 3x^2 - 2x}{x^4 (x^2 + 2x + 1)}$

$= \frac{x(-x^3 - 3x - 2)}{x^4 (x + 1)^2}$

so $\frac{dy}{dx} = - \frac{x^3 - 3x - 2}{x^3 (x + 1)^2}$

b) If $y = \frac{2x + 1}{3x^2 + 3x + 1}$.

We need to apply the quotient rule, as in previous example, so let

$u = 2x + 1$ and $v = 3x^2 + 3x + 1$, so

$\frac{du}{dx} = 2$ and $\frac{dv}{dx} = 6x + 3$, and

$\frac{dy}{dx} = \frac{(3x^2 + 3x + 1)(2) - (2x + 1)(6x + 3)}{(3x^2 + 3x + 1)^2}$

multiplying out the brackets

$\frac{dy}{dx} = \frac{6x^2 + 6x + 2 - 12x^2 - 6x - 6x - 3}{(3x^2 + 3x + 1)^2}$.

$= \frac{-(6x^2 + 6x + 1)}{(3x^2 + 3x + 1)^2}$