THE UNIVERSITY OF MELBOURNE

FACULTY OF BUSINESS & ECONOMICS

Helpsheet

Giblin Eunson Library

EXPONENTIAL AND LOG FUNCTIONS

Use this sheet to help you:

- Understand and apply the log laws
- Understand the application of the natural exponential and logarithmic functions
- Find the derivative of the natural exponential and logarithmic functions

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The exponential function $y = 10^x$

$y = 10^x$ is an example of an exponential function.

The key feature is that the independent variable, x, appears as the power (exponent) to which a constant (in this case 10) is raised.
The base does not need to be restricted to 10, in general, $\mathbf{y = a^x}$ where a is any constant greater than 1

The inverse function to $y = 10^x$

Given the function $y = 10^x$
The inverse function is defined as $x = \log_{10} y$.
To comply with convention we interchange the labels of the two variables and write the function as
$\mathbf{y = \log_{10} x}$

The following table gives a comparison of the logarithmic and index laws.

Log law	**Index law**
$\log(mn) = \log m + \log n$	$x^m \times x^n = x^{m+n}$
$\log\left(\frac{m}{n}\right) = \log m - \log n$	$\frac{x^m}{x^n} = x^{m-n}$
$\log 1 = 0$	$x^0 = 1$
$\log_a a = 1$	$x^1 = x$
$\log a^m = m\log^a$	

Practical uses of logs

Given $3^x = 91$, we want to find the value of x.
We cannot just use the y^x button on our calculator, as this can only be used when we know the value of x.
The method of solution is to take logs on both sides of the equation.
$\log(3)^x = \log 91$, now from our log rules $\log(3)^x = x\log 3$
$x\log 3 = \log 91$
dividing both sides by log3 gives (using the log key on your calculator)
$$x = \frac{\log 91}{\log 3} = \frac{1.9590}{0.4771} = 4.1061$$
This type of problem arises in economics when we want to solve for the number of years, x, in the compound growth formula $y = a(1 + r)^x$.

The natural exponential function

$y = e^x$.
Now as e is a constant, this is an exponential function. It is called the natural exponential function, because it was first developed to describe growth in nature, where plants and animals grow continuously.

Natural logarithms

$y = \log_e x$ is the natural logarithm function. It is commonly written as $y = \ln x$.
The same rules that applied to common logs also apply to natural logs.

Derivatives of exponential and logarithmic functions

The derivative of the natural exponential function is
If $y = e^x$, then $\frac{dy}{dx} = e^x$
If $y = e^{f(x)}$, then $\frac{dy}{dx} = e^x.f'(x)$, where $f'(x)$ denotes the derivative of $f(x)$.

e.g.1: if $y = e^{3x+1}$, where $f(x) = 3x + 1$, so
$f'(x) = 3$
then $\frac{dy}{dx} = 3e^{3x+1}$

e.g. 2: if $y = e^{-0.5x^2}$, where $f(x) = -0.5x^2$,
so $f'(x) = -x$
then $\frac{dy}{dx} = -xe^{-0.5x^2}$

The derivative of the natural logarithm function is
If $y = \ln x$, then $\frac{dy}{dx} = \frac{1}{x}$
If $y = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

e.g. 1: if $y = \ln(x^3 + 2x)$, where $f(x) = x^3 + 2x$,
so $f'(x) = 3x^2 + 2$
then $\frac{dy}{dx} = \frac{3x^2 + 2}{x^3 + 2x}$

e.g. 2: if $y = 2\log_e x$
then $\frac{dy}{dx} = \frac{2}{x}$